# A RIGOROUS EXPLICATION OF "EVENT MARKETS"

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Abstract. Like the weather, everyone talks about event markets. Like pornography, event marjets are hard to define – but everyone claims to know them when they see them.

## 1. Summary

The betting on the events that constitute Event Markets (EMs) are *not derivatives* no matter what certain legal contestants may claim. Although the events that comprise EMs share characteristics with various derivative offerings (futures, forwards, options), we maintain that this similarity is incidental as solely a reference to common platforms for contract execution, and decidedly outside of the scope of the Commodity Futures Trading Commission (CFTC) Rules and Authorization. Derivatives are about time to an expiry relative to an asset or liability value. Event markets are premised on the speculative use of knowledge using a liability over a time to expiry without reference to any underlying asset or liability. Event markets exist because of the *perception of knowledge imbalance*. As with all markets, a mathematical formulation is required. .

1.1. **Model.** We start by defining a *parameterized impulsive holding (PIH) $X_\theta$ with expiry $T$* to be a binomial random variable (RV) that will "assume" one of the values 0 or 2 when its expiry is reached. Implicit in this definition is the existence of a (measureable) parameter space $\Theta$ as well as a fixed "neutral" probability measure $\mathcal{P}_0$, defined on $\Theta$, with the property

$$\int_\Theta p(\{X_\theta = 0\})\, d\mathcal{P}_0(\theta) = \int_\Theta p(\{X_\theta = 2\})\, d\mathcal{P}_0(\theta) = \frac{1}{2}. \tag{1.1}$$

Note the following:

(1) To avoid introducing even more cumbersome notation, we denote by $p$ the conditional probability of $X_\theta$ when conditioned on $\theta \in \Theta$. These integrals represent marginalizations with respect to the joint distributions induced by the neutral probability;
(2) This need not be a "defining" characteristic of $\mathcal{P}_0$, as there may be many such distributions that possess this property;

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(3) Our definition of a PIH presumes the existence of at least one neutral probability measure. Note also that an expiry $T$ is similarly associated with every PIH: these are real, positive values that represent the point in the future at which the "resolution" of the event will occur. It is the presence of $T$ and the fact that these holdings resolve "instantaneously" when $T$ is reached that leads us to refer to these holdings as being "impulsive." We chose this term expressly to emphasize that the time span over which this change of state occurs is very short. We will have more to say about $T$ and its significance in what follows.

More concretely, the existence of the neutral probability is inferred because someone, somewhere, believes that *this distribution holds*, and hence the outcomes 0 and 2 are equally likely. In the EM setting, this mystical someone is the House, i.e., the entity that is offering the holding (original risk entertainment) through contract for purchase. But who will buy this holding, unless it has some kind of value, and whence does this value derive?

Here is where the role of the expiry $T$ enters: the assumption is that the holding can be purchased for 1 unit, and when the expiry is reached, the PIH RV will be sampled. The owner of the holding will then receive this realization, resulting in a net loss of her purchase price when the sampled value is 0, and net gain of 1 unit otherwise.

We now arrive at the crux of the matter: if the PIH RV is sampled from the distribution associated with the neutral distribution, then the expected value of the PIH is 1, and when one accounts for inherent transactional friction (e.g., commissions), this would result in a net negative return. Consequently, the only way it makes sense for anyone to purchase a PIH (ignoring any notion of non-quantifiable "entertainment value") is if she believes that *the neutral probability does not, in fact, hold*. More explicitly, we want to own a PIH precisely when we have a strong belief that some other probability measure, say $\mathcal{Q}_0$ is in force on $\Theta$, and that

$$\int_{\Theta} p(\{X_\theta = 2\})\, d\mathcal{Q}_0(\theta) > \frac{1}{2}. \tag{1.2}$$

Note that we have simplified things for the sake of expostion; to infer value, all that is required is an imbalance in the probability of the two outcomes with respect to $\mathcal{Q}_0$. This is due to the fact that there almost always exists another PIH $\bar{X}_\theta \triangleq 2 - X_\theta$ that is available for purchase, so that reversing the inequality in (1.2) for one would make the other the desirable holding. This leads directly to the observation that *the potential value of a PIH derives directly from the imbalance of knowledge that manifests itself as a difference between $\mathcal{P}_0$ and $\mathcal{Q}_0$*. While there are, in general, many ways to define the "difference" or "distance" between probability distributions, the current one is particularly salient: we perceive this difference through the lens of telling us how much money we feel we can make by exploiting it. PIHs, and therefore EMs themselves, are a tangible embodiment of *the value of knowledge*, and in fact, knowledge of a specifc kind, viz., that *which has limited time value and is, by construction, designed to expire*. In the next section, we explore this further, as well as address why we believe the current model, while simplistic, serves to capture the fundamental properties of those holdings that underlie EMs.

## 2. Discussion

Let's begin by addressing what could be construed as a misrepresentation: earlier, when introducing the alternative distribution $\mathcal{Q}_0$, we said that we would want to own a PIH "when we have a strong belief" that $\mathcal{Q}_0$ is the "true" distribution. Later, we spoke of the opportunity to own a PIH as representing "a tangible embodiment of the value of knowledge." Colloquially, one thinks of "belief" and "knowledge" as being, at best, first cousins; one can *believe* things without *knowing* them to be true: objective and subjective conditions of contract as attributes of assent are well understood. Consequently, in the vernacular, "knowing" surpasses (and certainly subsumes) "believing." In the current context, we maintain that a choice of distribution on $\Theta$ encapsulates entirely the state of knowledge about $\Theta$; its consequent effect on $X_\theta$ is captured by the conditional probability $p$. This then tells us everything that we can know about the ultimate fate of the holding. The state of *knowledge* is a probability distribution, while *learning* is the systematic evolution of this distribution over time.

We now move on to a discussion of the role played by time. The presence of the expiry $T$ makes this explicit insofar as we know that this is the point at which we will be given a realization of the PIH RV. (Note that it is an implicit assumption throughout that all parties, House, PIH holder, et al., measure time in the same way.) To simplify the exposition, we will assume that the sale of the PIH takes place at time $t = 0$, and hence we will use $T$ to represent both the point in time at which the PIH is sampled as well as the duration of time interval which will elapse from the time of the sale to the expiry. Again, we emphasize that the only reason we could have for purchasing the holding (at time $t = 0$) is that we are convinced that an alternative, non-neutral probability distribution $\mathcal{Q}_0$ exists, and so (1.2) holds.

It is possible that after the purchase of the PIH, niether we nor the House pay any attention to the PIH until the expiry is reached. At this point, we receive the realization of PIH RV and all our dealings with the House, insofar as these involve $X_\theta$, are concluded. However, it is also possible that our estimate of the alternative distribution changes over time; we can then consider

$$\mathcal{Q}_\cdot : [0, T) \to \mathfrak{P}(\Theta),$$

where $\mathfrak{P}(\Theta)$ represents the space of all probability measures on $\Theta$. Hence, we allow for the possibility that we *learn*, i.e., that the alternative probability distribution evolves over time. Of course, the House can learn too, and hence we must allow for the existence of $\{\mathcal{P}_t : t \in [0, T)\}$ representing the House's evolving state of knowledge. Note however that *none of the* $\mathcal{P}_t$*,* $t > 0$*, need be neutral.* We can use these to compute two real-valued functions of time, the PIH's value to the House, which we denote by $H_t$, and the value to us, i.e., the holding's owner, $O_t$.

$$H_t \triangleq C - \mathcal{E}_{\mathcal{P}_t} X_\theta$$

$$O_t \triangleq \mathcal{E}_{\mathcal{Q}_t} X_\theta - C$$

Here, $C$ represents the cost (consideration) of the contracted holding, i.e., that which we paid to the House to purchase the PIH, which is assumed to include commission. From our

discussion to this point, we see that $H_0 = C - 1 > 0$, while $O_0$ is also assumed to be greater than 0, or, once again, we would not have purchased the PIH. Their sum, $\Pi_t \triangleq H_t + O_t$ can be viewed as a measure of the degree to which the two sides' expectations vary over time due to a difference in their respective instantaneous states of knowledge; note that as long as $O_t > 0$ we (as owner of the PIH) expect profit.

When we introduce the signed measure $\pi_t \triangleq \mathcal{Q}_t - \mathcal{P}_t$ it is straightforward to show

$$\Pi_t = 2 \int_\Theta p(\{X_\theta = 2\})\, d\pi_t(\theta), \tag{2.1}$$

an expression which makes explicit the fact that this quantity depends entirely on the "knowledge gap." The knowledge gap can be considerate analogous to "chance" or "skill" as it is exists in the mind of beholder.

We observe at this point that the House can use $H_t$ as a barometer of (time varying) performance, and thus may be able to take actions before reaching the expiry to minimize its exposure. For example, if at some point $H_t$ attains an unacceptedly negative value, the House might offer the owner of the PIH a "buyout," and thereby cut its potential losses. Conversely, we would certainly make use of $O_t$ to decide whether such an offer was favorable before deciding to accept or reject it. In any case, because such considerations (on either/both sides) can occur, it is clear that the PIH has value throughout its lifetime, right up until its expiry is reached. $\Pi_t$ can be thought of as the instantaneous measure of the parties' relative states of knowlege: the greater this is in absolute value, the more disparate are these. In fact, this can also be viewed as a measure of the speculator's "exuberance": a positive value indicates that the owner is more confident than the House about the possibility of a "winning" outcome, while a negative value is indicative of the opposite. Moreover, such may be a sign that the PIH's owner is willing to entertain a buyout offer.

Continuing this line of reasoning, we note that we may model the act of sampling the PIH RV when $t = T$ as the calculation of expectation with respect to a probability measure that is supported entirely on regions in $\Theta$ defined by either $\{\theta \in \Theta : X_\theta = 0\}$ or $\{\theta \in \Theta : X_\theta = 2\}$. Because both parties must agree on which realization is achieved, it is reasonable to assume that the states of knowledge for both owner and House converge as $t \to T$. In addition, because by this point there is no gap in their knowedge, at least as it applies to the PIH, we must have $\Pi_T = 0$, i.e., there is no longer uncertainty about the outcome, and hence both parties expection of gain (resp., loss) align.

Due to the presence of an expiry in the definition of the PIH, it is clear why some descriptions of the events in an EM liken these to futures. In our opinion, this is ill-advised, as evidenced by the difficulty that arises when trying to identify wherein the value lies. Consider a standard call option: owning such clearly conveys upon the owner something that she values (again, else why would it be purchased). But in the case of the option in an exchange setting, there exists the presence of an underlying defined asset or liability that the owner of the option will have the right to purchase if it is financially advantageous to do so. (Alternatively, the option may reference an index that stands as a proxy for an underlying asset or liability.) It is the right to buy this security at a fixed price that gives

the option its value: no underlying asset, no value. In the EM setting, *there is no asset underlying the transaction.* As we have defined it, the PIH's value, or more accurately, the potential for profit, arises solely due to what we have termed the knowledge gap. We don't need to know how the House determined its initial neutral distribution, nor why it "cares" about which realization will be achieved at expiry. We simply wish to exploit, if possible, different states of knowledge.

Sporting events offer what can be considered canonical examples of PIHs. The House creates offerings that typically consist of the customer's picking the "winner" of some form of competition at brokered prices. Note that winning may be defined in various ways, but in non-money line wagers[1] it usually entails the use of some sort of adjustment factor (handicap, line, spread) when calculating the final "scores" that are accumulated by each competitor. The presumption is that the House relies on its knowledge of all the factors that might affect the outcome of the event to choose the adjustment factor appropriately: in the present setting, this means ensuring that either outcome is equally likely. A would-be bettor will purchase the associated PIH (i.e., make the wager) when she believes that her knowledge of these same factors indicates to her that these likelihoods are not, in fact, equal.

An interesting side note is that in such situations the expiry is not the start of the competition, but rather its end. This fact demonstrates that the expiry does not need to be a "fixed" time (e.g., 12:05 pm EDT, 1 May 2025) but simply an instant that both parties can agree has occurred. In addition, as the competition proceeds, both parties' states of knowledge continue to evolve, as do their respective valuations of the PIH, until at the conlusion they converge, yielding $\Pi_T = 0$.

Clearly, such a "sports contract" is not a future, forward or option. It is not used to hedge, risk manage or speculate on a defined "commodity." It does not assume a calendar periodicity or a common time. In an EM, the event possesses time value, and the value of the contract is a function of the underlying knowledge. Therefore, the role of time in an EM is to delimit the mutual desire to speculate on the outcome of an event *at a time*, not to hedge the value of a commodity *over time*. Similar examples of PIHs exist in many situations and are not limited to sporting events in an EM. Elections, going to the grocery story, purchasing tires – all are events with future expectations concerning price. Operating with an expectation of about *the* future, under the Commodity Exchange Act, does not transform it into *a* future.

## 3. Conclusions

An event in an EM is not a commodity, and it is not a derivative. Rather, it possesses primary market value based on differing states of knowledge between the House (i.e., market proprietor) and would-be buyer (customer). Hence, while it is possible to define quantities such as $H_0$ and $O_0$ as individual valuations ascribed to the House and owner, respectively,

[1] A money line wager is one where the non-zero outcome is not necessarily twice its purchase price. Everything would have proceeded *mutatis mutandis* if we had defined the PIH in this way initially; we removed this additional detail in order to simplify the exposition.

it is their difference as captured in $\Pi_0$ which is the salient quantity. In this, we note the similarity to physics, in particular the notion of *potential*: it is often the case that the actual value of the potential function is immaterial; what matters is the difference in potential. It is only when $\Pi_0$ is sufficiently greater than 0 (to account for the cost of the PIH) that a rational agent will make the purchase.

What we wish to stress is that the model proposed herein addresses and subsequently dismisses any concerns, given their ephemeral nature, about the "reality" of PIHs and any value that might be ascribed to them. The misclassification of EM component entities as futures, options or forwards (swaps) is confusing by intent; however, they are not derivatives despite the presence of the expiry. Potential for profit arises not due to the opportunity to obtain some underlying instrument (e.g., security) at a discount, but rather entirely due to the knowledge gap. As a consequence, the potential for profit disappears at the exact instant the width of that gap goes to zero.

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